Why did Alfvén oppose the use of his own frozen-in flux theorem?

The frozen-in flux theorem was put forward by Swedish physicist Hannes Alfvén. This theorem can be described as follows:

When the ideal conductive fluid moves in the magnetic field, the fluid element will freeze together with the magnetic field line, and the two can only move together. In other words, an ideal conductive fluid cannot cross the magnetic field line.

The title of frozen-in flux theorem in Wiki is Alfvén's theorem.

Alfvén said this about why frozen-in occurred:

"In view of the infinite conductivity, every motion (perpendicular to the field) of the liquid in relation to the lines of force is forbidden because it would give infinite eddy currents. Thus the matter of the liquid is 'fastened' to the lines of force…."

In his later years, Alfvén published an article [1] urging people to give up this theorem he found. The abstract of this article has only one sentence: "*It is shown that 'frozen-in magnetic field lines' and 'magnetic field-line reconnection' are unnecessary and often misleading concepts*".

Why is the frozen-in flux theorem unnecessary and misleading? Alfvén himself did not give a detailed explanation. Therefore, people continue to give great confidence to this theorem, especially in the field of plasma physics.

In recent years, we have carefully pondered this theorem and found that it is really a mistake. For this reason, we have published two SCI papers [2] [3], to discuss this problem.

We found that when deriving the frozen-in theorem, we only used the magnetic freezing equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad (1)$$

This is just the curl equation of the induced electric field. This equation alone is not enough to accurately describe the evolution of the electromagnetic system. At least the divergence equation of the induced electric field should be added.

$$\frac{q}{\epsilon_0} = -\nabla \cdot (\mathbf{u} \times \mathbf{B}) \qquad (2)$$

When an ideal conductor crosses the magnetic field, two situations will occur. Let's discuss it separately.

1，The induced electric field is irrotational everywhere

If the ideal conductor crosses the magnetic field along the isomagnetic surface, that is, moves along the path with zero magnetic gradient, the induced electric field inside it is irrotational everywhere.

This can be subdivided into three situations as shown in Figure 1, 2, and 3:

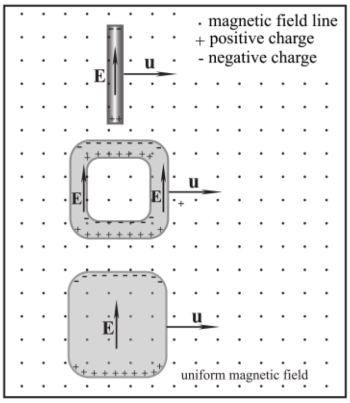

Figure 1. The ideal conductor crosses the magnetic field line in a uniform magnetic field.

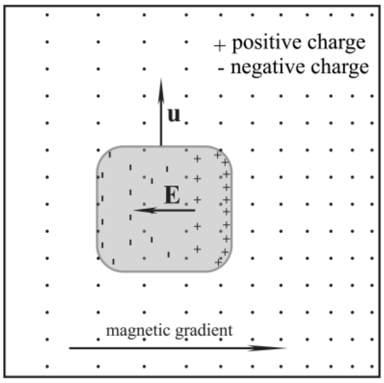

Figure 2. In a gradient magnetic field, the ideal conductor crosses the magnetic field line along the isomagnetic plane.

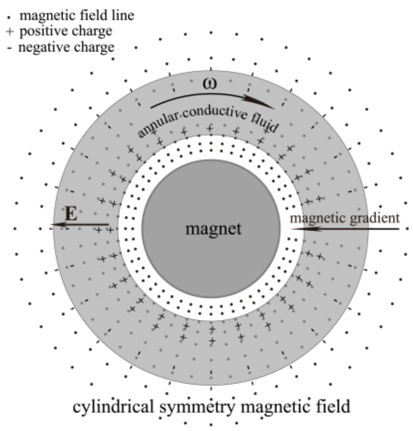

Figure 3. The ideal conductor rotates around the magnetic axis.

The above three conditions ensure that the internal magnetic flux is conserved everywhere.

It should be noted that in the above three cases, the internal induced electric field is **non-zero** and **irrotational**. In this way, although the conductivity is infinite, eddy current cannot occur. Since there will be no eddy current inside, there will be no magnetic resistance, because current is the necessary condition for magnetic resistance. Therefore, in the above three cases, the ideal conductor can freely cross the magnetic field lines without magnetic resistance. In our papers, this viewpoint has been proved by two kinds of experiments.

In the above three cases, the divergence of the internal induced electric field is not zero, and there will be polarization charges and polarization electric field. The distribution of electric charge and electric field are marked with positive and negative signs and arrows, which are determined by the right-handed rule. This shows that when the ideal conductor moves in the magnetic field, the charge will gather, some places have more positive charges, and some places have more negative charges. Macroscopically, it is no longer electrically neutral. This is very important. It will become very simple to explain the origin of sunspot magnetic field with this point.

We know that when the microscopic particles in the ideal conductor drift across the magnetic field, they follow a spiral path, which is due to the influence of Lorentz force. In the above three cases, the polarized electric field will always offset the Lorentz force generated by the movement of charged particles, so those charged particles no longer need to follow the spiral path, but directly follow the macroscopic movement path of the ideal conductor. For example, the macroscopic path of an ideal conductor is a straight line, and the microscopic particles in it also follow a straight line.

2. The induced electric field has a rotating component

If the magnetic gradient along the path direction is not zero when the ideal conductor crosses the magnetic field line, the magnetic flux passing through the ideal conductor will change. There will be a swirling induced electric field.

Will the swirling electric field cause infinite eddy currents (as Alfvén feared)? Unable! Because according to Lenz's law, the induced current is only to offset the change of external magnetic flux, and there will never be excess current and magnetic flux.

For example, as shown in Figure 4, when the ideal conductor is at the P_0 position, only two external magnetic lines pass through it. When it moves to the P_1 position, four external magnetic lines pass through it (two more magnetic field lines entered). According to Lenz's law, the inner part of the ideal conductor will automatically induce eddy current, which will produce two opposite endogenous magnetic lines to offset the change of the external magnetic flux. At the same time, the internal magnetic field interacts with the external magnetic field to form magnetic force. According to Lenz's law, the magnetic force is always opposite to the direction of motion of the conductor and always forms magnetic resistance (**F** in Figure 4).

Alfvén applied Ohm's law ($I = V/R$, or $I = \sigma E$) when considering the eddy current. He thought that the induced electric field multiplied by the conductivity is equal to the current density. In fact, he was wrong. Because Ohm's law is only applicable to static systems, not dynamic systems. In dynamic systems, the inductance and capacitance also play an important role. Once the resistivity is zero, the evolution of the system will be completely determined by capacitance, inductance, and Lenz's law.

As we all know, in the field of electrical engineering, the calculation of alternating current should not only consider resistance, but also consider inductive reactance and capacitive reactance, that is, use complex impedance instead of pure resistance. In the same way, on the problem of frozen-in, we can not only consider the resistance and ignore the inductance and capacitance, because we are facing a time-varying system rather than a stable system.

After considering the inductance, the singularity of infinite current is eliminated, and the reason for magnetic freezing proposed by Alfvén no longer exists.

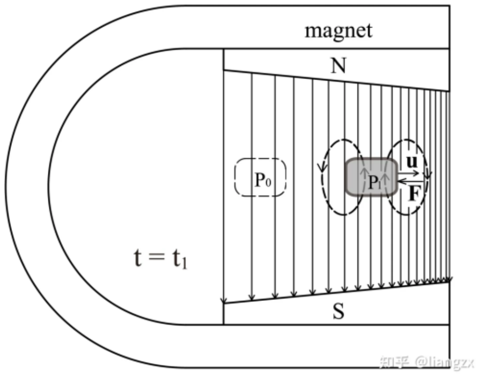

Figure 4. When the ideal conductor moves from P_0 to P_1, the endogenous magnetic flux (elliptical dotted line) appears.

From this example, we can see that the so-called conservation of magnetic flux is the total flux conservation of exogenous and endogenous magnetic flux, rather than the conservation of exogenous magnetic flux itself. Therefore, it is a key point to understand this problem to distinguish exogenous flux from endogenous flux and to find out which flux is conserved.

If the exogenous and endogenous magnetic field lines in Figure 4 are superimposed, it will look like Figure 5.

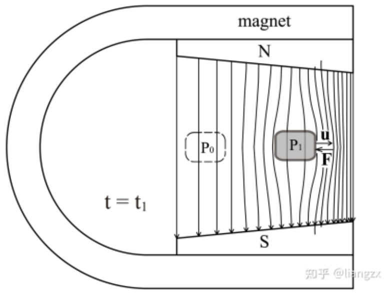

Figure 5. The appearance of the merging of endogenous and exogenous magnetic field lines.

Note the density distribution of the magnetic field lines after merging. The magnetic field lines in front of the conductor (right side) become denser due to pushing, which is obviously expected. But the magnetic lines behind it (on the left) also become denser. Is this unexpected? According to the frozen-in theorem, the magnetic field lines should look like Figure 6.

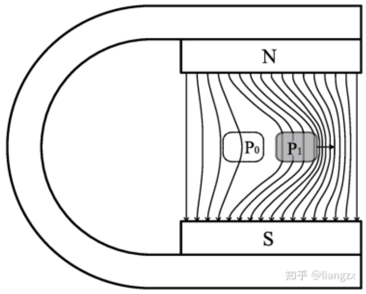

Figure 6. Magnetic field change given by frozen-in theorem.

If the frozen-in flux theorem is correct, when the ideal conductor moves from P_0 to P_1, it will also drag the magnetic field line to the right, which will inevitably make the magnetic line behind it thinner. At this point, our theory and frozen-in flux theorem give opposite predictions. Whose prediction is in line with the reality? This becomes an indicator to verify the correctness of the theories. We carried out experiments on this, and the results were published in our second paper. Experiments have proved that our prediction is correct, and the prediction given by the frozen-in flux theorem is wrong.

According to the frozen-in flux theorem, when an external force pushes an ideal conductor in the magnetic field, there will be a magnetic resistance due to dragging the frozen magnetic lines. However, what factors are related to the magnitude of magnetic resistance? There is no clear answer. The common saying is that the stronger the magnetic field, the greater the resistance.

With our theory, the magnetic resistance will be determined by two factors:

The magnitude of magnetic resistance is proportional to the change of exogenous magnetic flux (from P_0 to P_1 in Figure 4) and the magnetic gradient at P_1 point.

In short, the magnitude of magnetic resistance has nothing to do with the magnetic field strength, only with the magnetic field gradient. The latter group of experiments in our second paper has proved this viewpoint.

3. Relation with magnetic reconnection

Our theory has another advantage: if we understand the changes of exogenous and endogenous magnetic fields according to the image shown in Figure 4, the concept of "magnetic reconnection" is completely unnecessary.

Figure 7 shows the difference between the two theories in magnetic reconnection. If the frozen-in theorem is hold, when the ideal conductor is pulled far away from the magnetic field, the shape of the magnetic field line will not be understood, as shown in the upper right of Figure 7, unless magnetic reconnection is introduced. In our theory, as shown in Figure 4, the magnetic field is divided into two independent magnetic sources: internal magnetic source and external magnetic source. There is no phenomenon of dragging the magnetic field lines. Even if there is bending deformation of magnetic field lines, it is only the result of the superposition of internal and external magnetic fields. When the ideal conductor is pulled out of the magnetic field, as shown in the lower right of Figure 7, an independent conductor magnet will appear instead of magnetic reconnection.

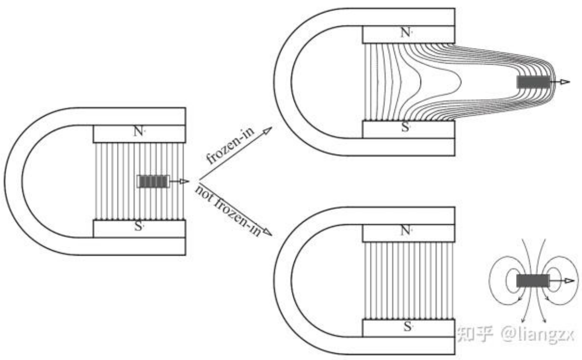

Figure 7. Difference between the two theories in magnetic reconnection.

Once there is no longer magnetic freezing and magnetic reconnection, some problems will become extremely simple.

For example, the rotation period of the sun near the equator is 25 days, but that near the pole is 35 days, which is called the differential rotation. The general magnetic field of the sun (the main dipole magnetic field) runs through the inside and outside the sun. If there is magnetic freezing, the differential rotation will twist its internal magnetic field lines ceaselessly, and the shape of the magnetic field lines will become indescribable. The magnetic resistance of twisting magnetic field lines should also make the differential rotation disappear gradually, and the rotation period at different latitudes tends to be consistent. However, this phenomenon did not appear.

If we accept that the magnetic field is not frozen, then the plasma at all latitudes on the sun can rotate independently, as shown in Figure 3, without affecting each other (ignoring friction). The problem of differential rotation will be greatly simplified, and the magnetic field like fried dough twist and the magnetic reconnection are no longer needed.

4. What is magnetic diffusion?

If the ideal conductor is replaced by a non-ideal conductor, magnetic diffusion will occur. There are different interpretations of magnetic diffusion between new and old theories:

In the old theory, magnetic diffusion means that some magnetic field lines escape from the conductor (or some magnetic field lines enter the conductor). Magnetic diffusion represents the phenomenon that the curvature of the magnetic field line and the magnetic potential energy are gradually reduced.

In our new theory, the magnetic diffusion represents the internal eddy current and endogenous magnetic flux disappear gradually.

In the old theory, you can't see the effect of internal eddy current. Because the old theory denied the induced electric field, hence it also denied the internal eddy current.

In our new theory, the function of eddy current is clear. The magnetic diffusion is just the process of the interaction between current and resistance to turn the endogenous magnetic energy into heat energy.

References

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